Calculating Two-Phase Pressure Drop

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Fluid flow concerns are quite prevalent among systems that handle chemicals, either in the liquid or the vapor state. An important parameter for characterizing the energy of the fluid flowing within a contained system, such as pipes, is pressure. This pressure becomes important for designing pipe sizes, determining pump requirements, and addressing safety concerns. Often the fluid is flowing as both liquid and gas.

Flow Type

The flow type of two-phase liquid-gas flow can be characterized into one of seven types shown in Figure 1. These types could be predicted using the following process parameters:

Gas density ratio $\rho \text{ (gas ratio)} = \rho \text{ (gas)} / \rho \text{ (air)}$

Liquid density ratio $\rho \text{ (liquid ratio)} = \rho \text{ (liquid)} / \rho \text{ (water)}$

Viscosity ratio $\mu \text{ (ratio)} = \mu \text{ (liquid)} / \mu \text{ (water)}$

Surface tension ratio $\sigma \text{ (ratio)} = \sigma \text{ (liquid)} / \sigma \text{ (water)}$

Mass Flux $\text{MF (liquid or gas)} = \text{F(liquid or gas)} / (3600 * \text{AR})$

To determine which type of flow exists, use the following coefficients in Figure 1, which is a flow-pattern plot.

$$\text{Y (lb/sec ft}^2) = \text{MF(gas)} / [\rho \text{ (gas ratio)} * \rho \text{ (liquid ratio)}]^{0.5}$$

$$\text{X} = \text{MF(liquid)} * [\rho \text{ (gas ratio)} * \rho \text{ (liquid ratio)}]^{0.5} * \mu \text{ (ratio)} / [\text{MF(gas)} * \sigma \text{ (ratio)}^3 * \rho \text{ (liquid ratio)}^2]$$

5-Step Pressure Drop Calculation

The following steps can be used. Three examples will be provided, two oil - hydrogen mixtures and one ethanol - air mixture. The following steps have been replicated in the downloadable MS Excel spreadsheet at http://www.rivercityeng.com.

Step 1: Select the pipe parameters.
When choosing the nominal pipe size, ensure that you have the appropriate inside diameter, based upon the pipe schedule. For the provided examples, a 4 inch standard pipe will be used for the oil - hydrogen mixtures and a 1 inch standard pipe will be used for the ethanol - air mixture. Both have an absolute roughness (ε) of 0.0018 inches.

Pipe Area $AR = (\pi * d^2) / 576$

Step 2: Obtain the process parameters.
The important properties are flow rate (F), safety factor (SF), density (ρ), viscosity (μ), and surface tension (σ). The first example has a 5,000 lb/hr flow, a 51.85 lb/ft^3 density, a 15 cP viscosity, and a 20 dynes / cm surface tension for the liquid (oil). The gas (hydrogen) is flowing at 800 lb/hr, with a 0.1420 lb/ft^3 density and a 0.012 cP viscosity. The second example is the same as the first, with the exception that the liquid flow is 140,000 lb/hr. The third example has a 158.8 lb/hr flow, a 61.3 lb/ft^3 density, a 1.07 cP viscosity, and a 51.4 dynes/cm surface tension for the liquid (ethanol). The gas (air) is flowing at 198.4 lb/hr, with a 0.0749 lb/ft^3 density and a 0.0181 cP viscosity. For all three examples, there is no safety factor (SF=1).

Step 3: Calculate the single phase line sizing pressure drop.
The following equations are used to calculate this pressure drop for both the liquid and the gas phase flow.

Velocity $v = F * SF / (3600 * \rho * AR)$

Reynolds Number $Re = 19.83 * F * SF / (\pi * d * \mu)$

Friction Factor $f = 64 / Re$ for $Re < 2100$

$f = 8 * [(8/Re)^{12} + 1/(A + B)^{1.5}]^{1/12}$

where $A = [2.457 * \ln(1 / ((7/Re)^{0.9} + 0.27 * \varepsilon / d))]^{16}$

$B = (37{,}530/Re)^{16}$

Pressure Drop $\Delta P = 4.167 * f * v^2 * \rho / (g_c * d)$ (this is in per 100 ft)

The first example has a 0.3 ft/sec velocity, a 523 Reynolds Number, a friction factor of 0.122 and a pressure drop of 0.02 psi / 100 ft of pipe for the liquid. The gas is flowing at 17.7 ft/sec, with a Reynolds Number of 105,000, a friction factor of 0.020 and a pressure drop of 0.03 psi / 100 ft. The second example has the same gas properties as the first example. However, the liquid is flowing at 8.48 ft/sec, with a Reynolds Number of 14,600, a friction factor of 0.029 and a pressure drop of 3.47 psi / 100 ft. The third example has a 0.12 ft/sec velocity, a 893 Reynolds Number, a friction factor of 0.072 and a pressure drop of 0.01 psi / 100 ft. The gas is flowing at 122.6 ft /sec, with a Reynolds Number of 66,000, a friction factor of 0.025 and a pressure drop of 3.53 psi / 100 ft.

Step 4: Calculate the two phase line sizing properties.
The density, velocity, and viscosity are averaged for a characteristic property of the combined phases in the fluid flow. And, the resulting two-phase Reynolds Number is calculated. The following equations are used:

Avg. density ρ (average) = (F(gas) + F(liquid))/(F(gas)/ ρ (gas) + F(liquid)/ ρ (liquid))

Avg. velocity v(average) = (F(gas) + F(liquid)) / (ρ (average) * AR)

Avg. viscosity μ (average) = (F(gas) + F(liquid))/(F(gas)/ μ (gas) + F(liquid)/ μ (liquid))

Figure 2 depicts values for this step and the next step for the examples provided.

Step 5: There are three different types of two-phase pressure drop correlations.
These are determined by the viscosity ratio and the mass flux.

a. For viscosity ratios greater than 1000 and a mass flux greater than 20.5, use the Chisholm-Baroczy (C-B) method [see example 1]. The C-B method is unique in that the pressure drops for each of the phases are calculated assuming that the total mixture flows as either liquid or gas. Therefore:

F(total) = F(liquid) +F(gas)

MF = F(total) / (3600*AR)

It should be noted that the Reynolds number and friction factor for each phase is also calculated assuming it is a function of total mass.

Re(liquid or gas) = f [F(total), SF, d, μ (liquid or gas)]
f (liquid or gas) = f [Re(liquid or gas), ε / d]

Δ P(liquid or gas) = 4.167 * f * MF^2 / (g_c * ρ(liquid or gas) * d) (this is in per 100 ft)

A pressure ratio is calculated:

$PR = [\Delta P(gas) / \Delta P(liquid)]^{0.5}$

Using this pressure ratio, a C-B constant is calculated:

$CB = 24.9 / MF^{0.5}$	for PR < 9.5
$CB = 235.3 / (PR * MF^{0.5})$	for 9.5 < PR < 28
$CB = 6788.5 / (PR^2 * MF^{0.5})$	for PR > 28

Now, the C-B pressure correction factor and the associated two-phase pressure drop is calculated:

$\varphi(C\text{-}B) = 1 + (PR^2 - 1) * (CB * (x_g^{((2-n)/2)}) * ((1-x_g)^{((2-n)/2)}) + x_g^{(2-n)})$

Where $x_g = F(gas) / (F(gas) + F(liquid))$ and $n=0.25$

$\Delta P\ (C\text{-}B) = 4.167 * \varphi\ (C\text{-}B) * f(liquid) * MF^2 / (g_c * \rho\ (liquid) * d)$ (this is in per 100 ft)

b. For viscosity ratios greater than 1000 and a mass flux less than 20.5, use the Lockhart - Martinelli (L-M) method [see example 2]. The Reynolds Number for both the liquid and the gas are used. Unlike the C-B method, the separate pressure drops for both the liquid and the gas are used explicitly, along with the pressure ratio. Using these, a unique L-M pressure correction factor for each phase is calculated. This requires the use of a different pressure factor than the C-B method:

$PR = \ln [(\Delta P\ (liquid) / \Delta P\ (gas))^{0.5}]$

b1. For Re(liquid) > 2100 and Re(gas) > 2100:

$\varphi\ (liquid) = 1.44-0.508*PR+0.0579*PR^2-0.000376*PR^3-0.000444*PR^4$
$\varphi\ (gas) = 1.44+0.492*PR+0.0577*PR^2-0.000352*PR^3-0.000432*PR^4$

b2. For Re(liquid) > 2100 and Re(gas) < 2100:

$\varphi\ (liquid) = 1.25-0.458*PR+0.067*PR^2-0.00213*PR^3-0.000585*PR^4$
$\varphi(gas) = 1.25+0.542*PR+0.067*PR^2-0.00212*PR^3-0.000583*PR^4$

b3. For Re(liquid) < 2100 and Re(gas) > 2100:

$\varphi\ (liquid) = 1.24-0.484*PR+0.072*PR^2-0.00127*PR^3-0.00071*PR^4$
$\varphi\ (gas) = 1.24+0.516*PR+0.072*PR^2-0.00126*PR^3-0.000706*PR^4$

b4. For Re(liquid) < 2100 and Re(gas) < 2100:

$\varphi\ (liquid) = 0.979-0.444*PR+0.096*PR^2-0.00245*PR^3-0.00144*PR^4$
$\varphi\ (gas) = 0.979+0.555*PR+0.096*PR^2-0.00244*PR^3-0.00144*PR^4$

Now, a separate pressure drop is calculated for each phase:

$\Delta P(liquid1) = [\exp[\varphi\ (liquid)]]^2 * \Delta P\ (liquid)$
$\Delta P(gas1) = [\exp[\varphi\ (gas)]]^2 * \Delta P\ (gas)$

Then, the estimated two-phase pressure drop is the maximum of these:

$\Delta P(L\text{-}M) = \max \{ \Delta P(liquid1), \Delta P\ (gas1)\}$

c. Finally, for viscosity ratios less than 1000, use the Friedel method [see example 3]. In addition to the Reynolds number, the Froude number and Weber numbers are used. In the following equations, be sure to use the mass flux of the total mass (liquid+gas) flowing in the pipe.

Froude Number $Fr = 12 * MF^2 / (g_c * \rho\,(\text{average})^2 * d)$

Weber Number $We = 37.8 * d * MF^2 / (\rho\,(\text{average}) * \sigma)$

A gas mass ratio and two Friedel coefficients are also used:

Gas mass ratio $x_g = F(\text{gas}) / [F(\text{liquid}) + F(\text{gas})]$

Calculate $\xi 1$ for both phases using the Reynolds number calculated for each phase:

Friedel coefficient 1 $\xi 1 = 64 / Re$ for $Re < 1055$

$\xi 1 = [0.86859 * \ln(Re/(1.964*\ln(Re) - 3.8215))]^{-2}$ for $Re > 1055$

Friedel coefficient 2 $\xi 2 = (1 - x_g)^2 + (x_g{}^2) * [\rho\,(\text{liquid}) * \xi 1(\text{gas}) / \{\rho\,(\text{gas}) * \xi 1(\text{liquid})\}]$

Two pressure correction factors are used, one for horizontal flow (which includes vertical up) and another for vertical down flow. The correlation for horizontal flow is:

$$\varphi\,(F) = \xi 2 + 3.24 * x_g{}^{0.78} * (1 - x_g)^{0.24} * (\rho\,(\text{liquid})/\rho\,(\text{gas}))^{0.91} * (\mu\,(\text{gas})/\mu\,(\text{liquid}))^{0.19} * (1 - \mu\,(\text{gas})/\mu\,(\text{liquid}))^{0.70} * Fr^{-0.045} * We^{-0.035}$$

The correlation for vertical down flow is:

$$\varphi\,(F) = \xi 2 + 38.5 * x_g{}^{0.75} * (1 - x_g)^{0.314} * (\rho\,(\text{liquid})/\rho\,(\text{gas}))^{0.86} * (\mu\,(\text{gas})/\mu\,(\text{liquid}))^{0.73} * (1 - \mu\,(\text{gas})/\mu\,(\text{liquid}))^{6.84} * Fr^{-0.0001} * We^{-0.037}$$

Finally, the two-phase pressure drop is then calculated using the following:

$\Delta P(F) = 4.167 * \varphi\,(F) * \xi 1(\text{liquid}) * MF^2 / (g_c * \rho\,(\text{liquid}) * d)$ (this is in per 100 ft)

Symbol List

Symbol	Definition	Units
A	Friction Factor number	none
AR	Pipe Area	ft2
B	Friction Factor number	none
CB	Chisholm-Baroczy constant	dimensionless
d	Pipe Diameter	inch
f	Friction Factor	none
Fr	Froude number	none
g_c	Dimensional constant	32.174 lb ft / lbf sec^2
F	Mass flow	lb / hour
MF	Mass flux	lb/ft2-sec
n	Chisholm-Baroczy constant	Dimensionless
P	Pressure	Psi
PR	Pressure ratio	dimensionless
Re	Reynolds number	none
SF	Safety factor (none = 1)	dimensionless
v	Velocity	feet / second
We	Weber number	none
x_l	Liquid Mass Ratio	none
x_g	Gas Mass Ratio	none
X	Dimensionless Flow Pattern Region coefficient	none
Y	Flow Pattern Region coefficient	pounds / sec ft^2
ε	Surface Roughness	inches
ξ	Friedel coefficient	none
φ	Pressure Correction factor	none
ρ	Density	lb / ft^3
σ	Surface Tension	dynes / centimeter
μ	Viscosity	Centipoise

Figure 1: Flow Types

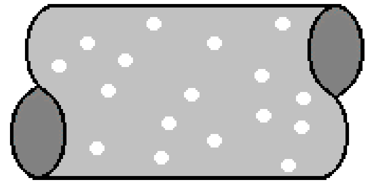
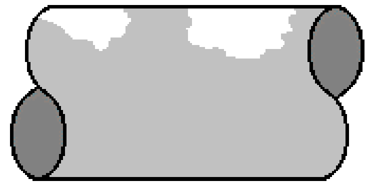
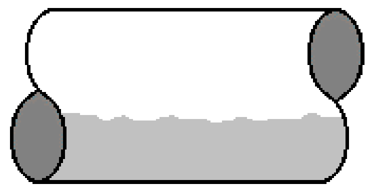
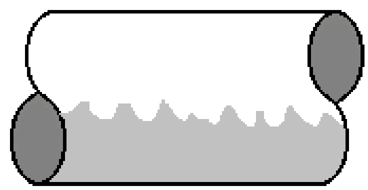
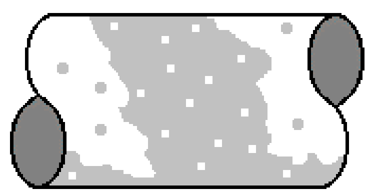
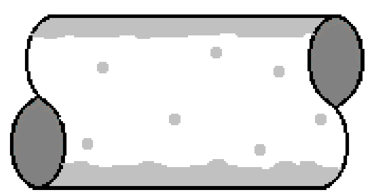
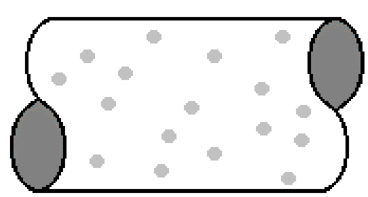
Type	Description	Sketch
Bubble (froth)	Liquid with dispersed bubbles of gas.	
Plug	alternating plugs of gas and liquid in upper section of pipe.	
Stratified	Liquid on the lower and gas on the upper section pipe separated by a smooth interface.	
Wave	Same as stratified, except separated by a wavy interface traveling in the same direction of flow.	
Slug	Similar to stratified, except the gas periodically picks up a wave and forms a bubbly plug. This flow can cause severe and dangerous vibrations because of the impact of the high-velocity slugs against the equipment.	
Annular	Gas in the center and liquid on the outer portion of the pipe.	
Spray (dispersed)	Liquid droplets in the gas.	

Figure 2: Example Results

	Example		
Property	**1**	**2**	**3**
Average Density	1.012	16.895	0.135
Average Velocity	18.01	26.19	122.72
Average Viscosity	0.087	1.853	0.032
Reynolds Number	105,000	119,000	66,900
Viscosity Ratio	1250	1250	59
Mass Flux	18	442	17
Correlation Type	L-M	C-B	Friedel
Pressure Drop			
Horizontal	0.28	9.64	9.86
Vertical down			11.10

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